Date: 05/05/2022

Question Paper Code

430/2/1

Time: 2 Hrs.

Class-X

Max. Marks: 40

MATHEMATICS (Basic) Term-II

(CBSE 2022)

GENERAL INSTRUCTIONS

(i) *This question paper contains **14** questions. **All** questions are compulsory.*

(ii) *This question paper is divided into 3 Sections – **Section A, B** and **C**.*

(iii) ***Section-A** comprises of **6** questions (Q. Nos. **1** to **6**) of **2** marks each.*

*Internal choice has been provided in **two** questions.*

(iv) ***Section-B** comprises of **4** questions (Q. Nos. **7** to **10**) of **3** marks each.*

*Internal choice has been provided in **one** question.*

(v) ***Section-C** comprises of **4** questions (Q. Nos. **11** to **14**) of **4** marks each. An internal choice has been provided in **one** question. It also contains **two** case study based questions.*

(vi) *Use of calculator is not permitted.*

SECTION-A

Question Numbers 1 to 6 carry 2 marks each.

1. Find the nature of the roots of the quadratic equation : **[2]**

   $4x^2 - 5x - 1 = 0$

Solution

$4x^2 - 5x - 1 = 0$

$D = b^2 - 4ac$, where $a = 4$, $b = -5$ and $c = -1$ [½]

$\Rightarrow$ $D = 25 + 16 = 41$ [½]

$\Rightarrow$ $D > 0$ [½]

$\therefore$ The given equation has real and distinct roots [½]

2. **(a)** Which term of the A.P. 3, 8, 13, 18, … is 78? **[2]**

OR

(b) Find the common difference of an A.P. whose n^{th} term is given by **[2]**

$a_n = 6n - 5$.

Solution

(a) Given A.P. is 3, 8, 13, 18, … .

Here, $a = 3$ and $d = 8 - 3 = 5$ [½]

$a_n = a + (n - 1)d$ [n^{th} term] [½]

$\Rightarrow$ $78 = 3 + (n - 1)5$

$\Rightarrow$ $\frac{75}{5} = n - 1$ [½]

$\Rightarrow$ $n = 16$

$\therefore$ 78 is 16^{th} term of the given A.P. [½]

OR

(b) n^{th} term of A.P. is

$a_n = 6n - 5$

if $n = 1$,

$\Rightarrow$ $a_1 = 6 - 5 = 1$ [½]

if $n_2 = 1$,

$a_2 = 6 \times 2 - 5 = 7$ [½]

$\therefore$ Common difference $(d) = 7 - 1$ [½]

$= 6$ [½]

3. 3 cubes each of 8 cm edge are joined end to end. Find the total surface area of the cuboid so formed. **[2]**

Solution

Dimensions of cuboid are 24 cm, 8 cm, 8 cm

T.S.A of cuboid $= 2(lb + bh + lh)$ [½]

$= 2[24(8) + 8(8) + 24(8)]$ [½]

$= 2[192 + 64 + 192]$ [½]

$= 2[448] = 896 \text{ cm}^2$ [½]

4. (a) In Fig. 1, perimeter of ΔPQR is 20 cm. Find the length of tangent *PA*. **[2]**

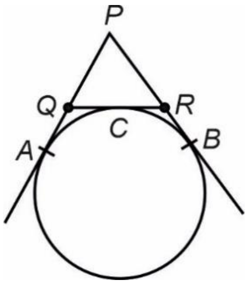


Fig. 1

OR

(b) In Fig. 2, *BC* is tangent to the circle at point *B* of circle centred at *O*. *BD* is a chord of the circle so that $\angle BAD = 55°$. Find $m\angle DBC$. **[2]**

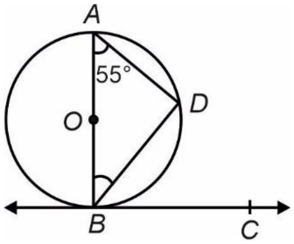
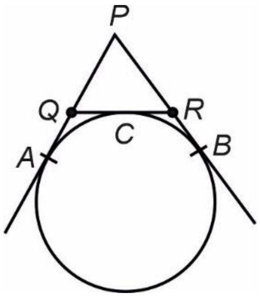


Fig. 2

Solution

(a)



$PA = PQ + QA$

$= PQ + QC$ ...(i) [$\because$ $QA = QC$] **[½]**

and $PB = PR + BR$ **[½]**

$= PR + CR$ ...(ii) [$\because$ $BR = CR$]

Adding (i) and (ii), we get

$PA + PB = PQ + QC + CR + PR$

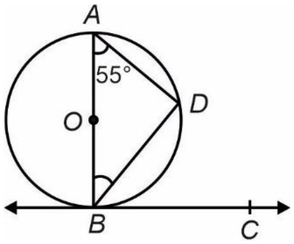
$\Rightarrow$ $2PA = PQ + QR + PR$ [$\because$ $PA = PB$] **[½]**

$\Rightarrow$ $PA = \dfrac{\text{Perimeter of } \Delta PQR}{2}$

$= \dfrac{20}{2}$ **[½]**

$= 10$ cm

OR

(b)


$\angle ADB = 90°$ [Angle in semi-circle] **[½]**

$\angle ABD = 90° - \angle BAD$ [Angle sum property of ΔABD]

$= 90° - 55°$

$= 35°$ **[½]**

Now, $\angle DBC = 90° - \angle ABD$ [$\because$ $AB \perp BC$] **[½]**

$= 90° - 35°$

$= 55°$ **[½]**

5. Find the mode of the following frequency distribution: **[2]**

Class :	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency :	25	30	45	42	35

Solution

$$\text{Mode} = l + \frac{(f_m - f_1)}{(2f_m - f_1 - f_2)} \times h$$ **[½]**

$\Rightarrow$ $f_m = 45$

$f_1 = 30$

$f_2 = 42$

$h = 10$ **[½]**

$l = 40$

$\therefore$ $\text{Mode} = 40 + \left(\frac{45 - 30}{90 - 72}\right) \times 10$ **[½]**

$= 40 + \left(\frac{15}{18} \times 10\right) = 40 + \left(\frac{150}{18}\right) = 40 + 8.33 = 48.33$ **[½]**

6. Find the sum of the first fifteen multiples of 8. **[2]**

Solution

First fifteen multiples of 8 are

8, 16, 24….. **[½]**

Here, $a = 8$ and $d = 8$

$S_{15} = \frac{15}{2}[2 \times 8 + (15 - 1)8]$ **[½]**

$= \frac{15}{2}[16 + 112]$ **[½]**

$= \frac{15 \times 128}{2}$

$= 960$

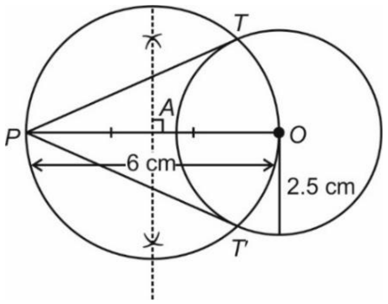
$\therefore$ Sum of first fifteen multiples of 8 is 960. **[½]**

SECTION-B

Question Numbers 7 to 10 carry 3 marks each.

7. Draw a circle of radius 2.5 cm. Construct a pair of tangents from a point *P* at a distance of 6 cm from the centre of the circle. **[3]**

Solution



$\therefore$ *PT* and *PT'* are the required tangents. **[3]**

8. (a) As observed from the top of a light house 100 m above sea level, the angle of depression of a ship, sailing directly towards it, changes from 30° to 45°. Determine the distance travelled by the ship during this time. **[3]**

$\left(\text{Use } \sqrt{3} = 1.73\right)$

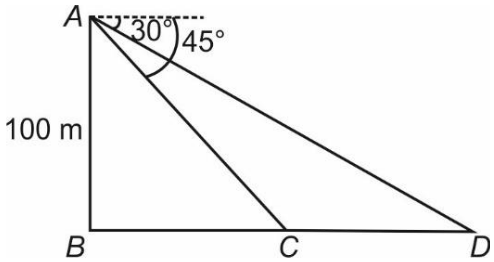
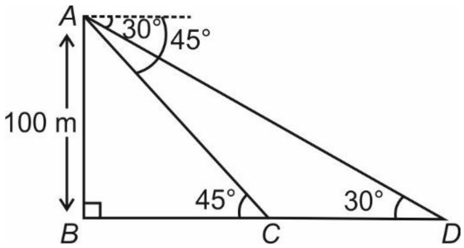


Fig. 3

Solution



In ΔABC,

$$\frac{AB}{BC} = \tan 45° = 1$$ **[½]**

$\Rightarrow$ $AB = BC = 100$ m ...(i) **[½]**

In ΔABD,

$$\frac{AB}{BD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $BD = AB \times \sqrt{3}$

$= 100\sqrt{3}$ m ...(ii) **[½]**

$\therefore$ $CD = BD - BC$

$= \left(100\sqrt{3} - 100\right)$ m [From (i) and (ii)] **[½]**

$= 100\left(\sqrt{3} - 1\right)$ m

= 100 (1.73 –1) m

= 100 × 0.73 m **[½]**

= 73 m

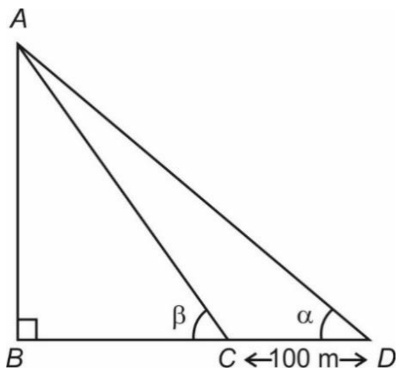
$\therefore$ Ship will travel 73 m during the given time. **[½]**

OR

(b) At a point on level ground, the angle of elevation of a vertical tower is, found to be α such that $\tan\alpha = \frac{1}{3}$.

After walking 100 m towards the tower, the angle of elevation β becomes such that $\tan\beta = \frac{3}{4}$. Find the height of the tower. **[3]**

Solution



Let *AB* represents the tower. Observer is moving from *D* to *C*.

In ΔABC,

$$\tan\beta = \frac{AB}{BC} = \frac{3}{4} \quad \text{...(i)}$$ **[½]**

and in ΔABD,

$$\tan\alpha = \frac{AB}{BD} = \frac{1}{3} \quad \text{...(ii)}$$ **[½]**

From (i) and (ii), we get

$$BC = \frac{4AB}{3} \text{ and } BD = 3AB$$ **[½]**

$\Rightarrow$ $CD = BD - BC$ **[½]**

$\Rightarrow$ $100 = 3AB - \frac{4AB}{3}$

$\Rightarrow$ $100 = \frac{9AB - 4AB}{3}$ **[½]**

$\Rightarrow$ 300 = 5*AB*

$\Rightarrow$ *AB* = 60 m

$\therefore$ Height of tower is 60 m. **[½]**

9. Find the mean of the following frequency distribution : **[3]**

Class :	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
Frequency :	4	10	5	6	5

Solution

Class	**Frequency (f_i)**	**Class Marks (x_i)**	**Product (f_ix_i)**
10–15	4	12.5	50.00
15–20	10	17.5	175.00
20–25	5	22.5	112.50
25–30	6	27.5	165.00
30–35	5	32.5	162.50
Total	***N* = 30**		$\sum f_i x_i =$ **665.00**

[1]

Mean $(\bar{x}) = \frac{1}{N}\sum_{i=1}^{k} f_i x_i$ **[1]**

$= \frac{\sum_{i=1}^{5} f_i x_i}{N} = \frac{665.0}{30}$ **[1]**

= 22.17 (approx.)

10. The median of following frequency distribution is 25. Find the value of *x*. **[3]**

Class :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency :	6	9	10	8	*x*

Solution

Class	**Frequency**	**c.f.**
0–10	6	6
10–20	9	15
20–30	10	25
30–40	8	33
40–50	*x*	33+*x*

[½]

Median = 25

$\Rightarrow$ Median class is 20–30

$\Rightarrow$ $f = 10$, c.f. = 15, $N = 33 + x$, $h = 10$ and $l = 20$ **[½]**

Median = $l + \left(\frac{\frac{N}{2} - \text{cf}}{f}\right) \times h$ **[½]**

$\Rightarrow \quad 25 = 20 + \left(\frac{\frac{33+x}{2} - 15}{10} \times 10\right)$ **[½]**

$\Rightarrow \quad 5 = \dfrac{33 + x - 30}{2}$ **[½]**

$\Rightarrow \quad 10 = 3 + x$

$\therefore \quad x = 7$ **[½]**

SECTION-C

Question Numbers 11 to 14 carry 4 marks each.

11. (a) Prove that a parallelogram circumscribing a circle is a rhombus. **[4]**

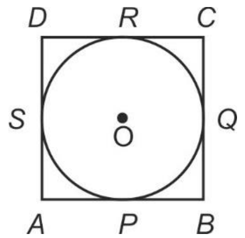
OR

(b) Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle. **[4]**

Solution (a)

Given : A circle with centre *O*.

A parallelogram *ABCD* touching the circle at Points *P*, *Q*, *R* and *S*



To Prove: *ABCD* is a rhombus

Proof: A rhombus is a parallelogram with all sides equal

In parallelogram *ABCD*

$AB = CD$ and $BC = AD$ **[1]**

We know that the lengths of tangents from an external point are equal

$\therefore \quad AP = AS \quad \ldots(i)$

$BP = BQ \quad \ldots(ii)$

$CQ = CR \quad \ldots(iii)$

$DR = DS \quad \ldots(iv)$ **[1]**

Adding (i), (ii), (iii) and (iv), we get

$\Rightarrow AP + BP + CR + DR = AS + BQ + CQ + DS$

$\Rightarrow AB + (CR + DR) = AS + BQ + CQ + DS$

$\Rightarrow AB + CD = (AS + DS) + (BQ + CQ)$

$\Rightarrow AB + CD = AD + BC$

$\Rightarrow CD + CD = BC + BC$

$[\because AB = CD$ and $AD = BC]$ **[1]**

$\Rightarrow CD = BC$

$\therefore AB = CD = BC = AD$

All sides are equal

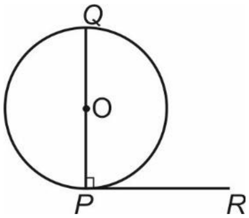
$\Rightarrow$ Hence, *ABCD* is a rhombus **[1]**

OR

Solution (b)

Let, O is the centre of the given circle. A segment PR has been drawn touching the circle at point P. **[½]**

Draw $QP \perp RP$ at point P, such that point Q lies on the circle. **[½]**



$\angle OPR = 90°$ [Radius ⊥ Tangent] **[½]**

Also, $\angle QPR = 90°$ [given] **[½]**

$\therefore \angle OPR = \angle QPR$ **[½]**

Now, the above case is possible only when centre O lies on the line QP. **[1]**

Hence, perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle. **[½]**

12. The sum of the ages of a boy and his sister (in years) is 25 and product of their ages is 150. Find their present ages. **[4]**

Solution

Let age of boy be x years, then age of his sister will be $(25 - x)$ years **[½]**

Product of their ages, $(x)(25 - x) = 150$ **[½]**

$\Rightarrow 25x - x^2 = 150$ **[½]**

$\Rightarrow x^2 - 25x + 150 = 0$ **[½]**

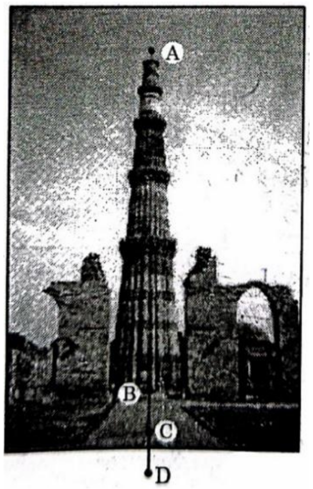
$\Rightarrow (x - 15)(x - 10) = 0$ **[1]**

$\Rightarrow x = 10$ and 15 **[½]**

Hence, their present age's are 10 years and 15 years. **[½]**

Case Study - 1

13. Qutub Minar, located in South Delhi, India, was built in the year 1193. It is 72 m high tower. Working on a school project, Charu and Daljeet visited the monument. They used trigonometry to find their distance from the tower. Observe the picture given below. Points C and D represent their positions on the ground in line with the base of tower, the angles of elevation of top of the tower (Point A) are 60° and 45° from points C and D respectively.

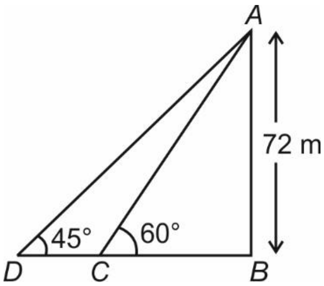


(i) Based on above information, draw a well-labelled diagram. **[1]**

(ii) Find the distances *CD*, *BC* and *BD*. (use $\sqrt{3}$ = 1.73) **[3]**

Solution

(i) Let positions of Charu and Daljeet be *C* and *D* respectively, **[1]**



Charu is nearer to Qutub Minar as its angle of elevation is greater.

(ii) In $\triangle ABC$,

$$\tan 60° = \frac{AB}{BC}$$ **[½]**

$$\Rightarrow \sqrt{3} = \frac{72}{BC}$$ **[½]**

$\Rightarrow BC$ = 41.52 m **[½]**

In $\triangle ABD$,

$$\tan 45° = \frac{AB}{BD}$$ **[½]**

$$\Rightarrow 1 = \frac{72}{BD}$$

$\Rightarrow BD$ = 72 m **[½]**

$CD = BD - BC$

CD = (72 – 41.52) m

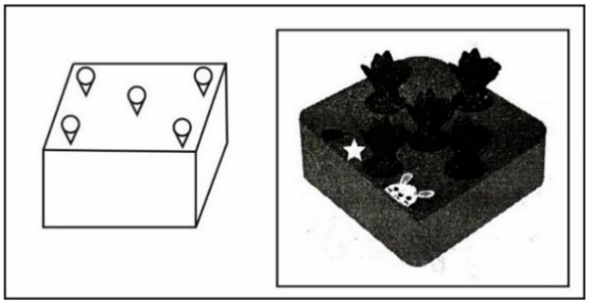
= 30.48 m **[½]**

Case Study - 2

14. A solid cuboidal toy is made of wood. It has five cone shaped cavities to hold toy carrots.

The dimensions of the toy are cuboid – 10 cm × 10 cm × 8 cm.

Each cone carved out – Radius = 2.1 cm and Height = 6 cm.



(i) Find the volume of wood carved out to make five conical cavities. **[2]**

(ii) Find the volume of the wood in the final product. **[2]**

Solution

(i) Dimensions of cuboid = 10 cm × 10 cm × 8 cm

Dimensions of cone,

Radius, R = 2.1 cm

Height, H = 6 cm

Volume of wood carved out = Volume of 5 cones = $\frac{1}{3}(\pi)R^2H \times 5$ **[1]**

$= 5 \times \frac{1}{3} \times \frac{22}{7} \times (2.1)^2 \times 6 = 138.6 \text{ cm}^3$ **[1]**

(ii) Volume of the wood in the final product = Volume of cuboid – Volume of wood carved out **[1]**

= $(10 \times 10 \times 8 - 138.6) \text{ cm}^3$ **[½]**

= 661.4 cm^3 **[½]**

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